Antiderivative Antialiasing with Frequency Compensation for Stateful Systems

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Main idea

Antiderivative antialiasing (AA) techniques [1, 2] turn a static nonlinearity

$$y_n = f(x_n)$$

into a dynamic nonlinear process. In linear terms, the frequency response is affected. We want to add a compensation filter in series to restore the original frequency response and substitute static nonlinearities in stateful systems locally and without affecting the overall frequency response and stability, unlike previous methods [3, 4].

AA-FIR

First-order AA-FIR [1]:

$$y_n = \begin{cases} \dfrac{F(x_n) - F(x_{n-1})}{x_n - x_{n-1}} & x_n \neq x_{n-1} \\ f(\bar{x}) & x_n = x_{n-1} = \bar{x} \end{cases},$$

where F is the antiderivative of f. Linearazing:

$$\widetilde{y}_n = f'(0)\frac{x_n + x_{n-1}}{2},$$

which is not minimum phase, and hence not invertible. We get similar results at higher orders.

AA-IIR

We can apply AA-IIR [2] using any IIR filter

$$H(s) = A_0 + \sum_{k=1}^{p}\sum_{l=1}^{m_k} \frac{A_{kl}}{(s-\alpha_k)^l} + \sum_{k=1}^{q}\sum_{l=1}^{m_k}\left(\frac{B_{kl}}{(s-\beta_k)^l} + \frac{\overline{B}_{kl}}{(s-\overline{\beta}_k)^l}\right).$$

Simple real pole

Suppose that $H(s) = \frac{A}{s-\alpha}$, then applying AA-IIR

$$y_n = e^{\alpha} y_{n-1} + A\int_0^1 f(x_{n-1} + t(x_n - x_{n-1}))e^{\alpha(1-t)}\,\mathrm{d}t,$$

and in linear terms

$$\widetilde{y}_n = e^{\alpha}\widetilde{y}_{n-1} + f'(0)\frac{A}{\alpha^2}\left((e^{\alpha} - \alpha - 1)x_n + ((\alpha - 1)e^{\alpha} + 1)x_{n-1}\right),$$

whose zero

$$\zeta = -\frac{(\alpha-1)e^{\alpha}+1}{e^{\alpha}-\alpha-1}, \qquad \alpha < 0 \rightarrow -1 < \zeta < 0.$$

Higher-order terms

- **Multiple real pole**: unstable for all useful α values;
- **Simple complex conjugate poles**: several areas of instability on the s plane, containing the useful regions;
- **Multiple complex conjugate poles**: hard to study but it seems analogous to the previous case;
- **In general**: impossible to study stability systematically.

Effects of reconstruction process

Both AA-FIR and AA-IIR use linear interpolation for input reconstruction. Using nearest neighbor interpolation, instead, makes areas of instability shrink sensibly in multiple real pole AA-IIR.

Hence, the reconstruction process has an important effect on the stability of the compensation filter and should be researched further.

Effects of numerical integration

Employing numerical methods to compute integrals in AA-IIR causes minor changes on the areas of instability and DC gain of the compensating filter. In other words, it should not make a big difference in practice.

Case study: the diode clipper circuit

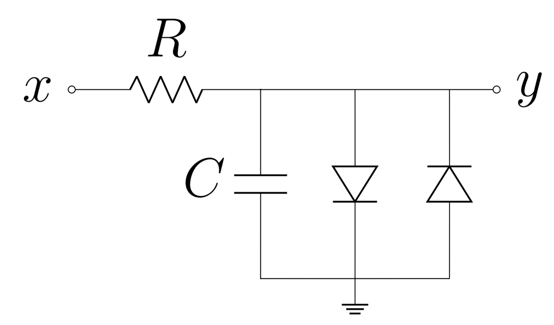

Figure 1: Schematics of the diode clipper circuit.

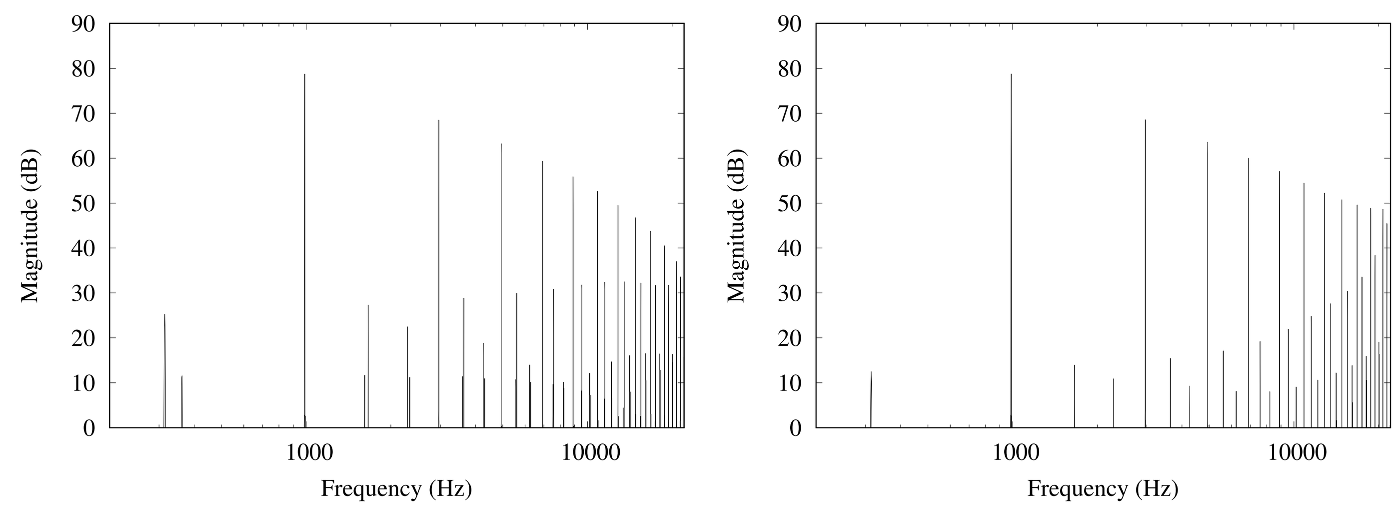

Figure 2: Backward Euler method, (left) original and (right) modified algorithm.

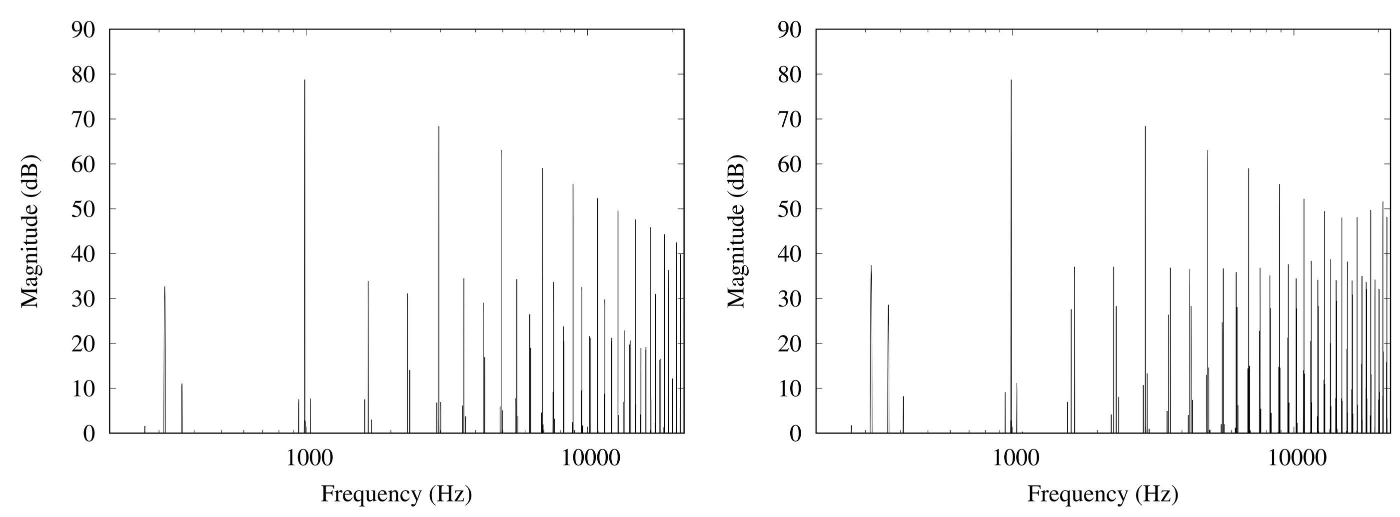

Figure 3: Bilinear transform, (left) original and (right) modified algorithm.

Discretization choice affects aliasing and antialiasing performance.

References

[1] J. Parker, V. Zavalishin, and E. Le Bivic, "Reducing the aliasing of nonlinear waveshaping using continuous-time convolution," in *Proc. 19th Intl. Conf. Digital Audio Effects (DAFx-16)*, (Brno, Czech Repulic), pp. 137–144, 2016.

[2] P. P. La Pastina, S. D'Angelo, and L. Gabrielli, "Arbitrary-order IIR antiderivative antialiasing," in *Proc. 24th Intl. Conf. Digital Audio Effects (DAFx20in21)*, (Vienna, Austria), pp. 9–16, 2021.

[3] M. Holters, "Antiderivative antialiasing for stateful systems," in *Proc. 22nd Intl. Conf. Digital Audio Effects (DAFx-19)*, (Birmingham, UK), 2019.

[4] M. Holters, "Antiderivative antialiasing for stateful systems," *Applied Sciences*, vol. 10, no. 1, 2020.