A QUADRIC SURFACE MODEL OF VACUUM TUBES FOR VIRTUAL ANALOG APPLICATIONS

Riccardo Giampiccolo [1], *Stefano D'Angelo* [2], *Alberto Bernardini* [1], *and Augusto Sarti* [1]

[1] Dipartimento di Elettronica, Informazione e Bioingegneria (DEIB), Politecnico di Milano, Milan, Italy

[2] Orastron srl, Agropoli, Italy

1 - Abstract

Despite the prevalence of modern audio technology, vacuum tube amplifiers continue to play a vital role in the music industry. For this reason, over the years, many different digital techniques have been introduced for accomplishing their emulation. In this paper, we propose a novel quadric surface model for tube simulations able to overcome the Cardarilli model in terms of efficiency whilst retaining comparable accuracy when grid current is negligible. After showing the model capability to well outline tubes starting from measurement data, we perform an efficiency comparison by implementing the considered tube models as nonlinear 3-port elements in the Wave Digital domain. We do this by taking into account the typical common-cathode gain stage employed in vacuum tube guitar amplifiers. The proposed model turns out to be characterized by a speedup of $4.6\times$ with respect to the Cardarilli model, proving thus to be promising for real-time Virtual Analog applications.

3 - Proposed Quadric Surface Model

We propose a new memoryless triode model based on a quadric surface approximation. The model is characterized by a lower number of parameters and by simpler mathematical operations with respect to those involved in the two traditional models. On the other hand, for the sake of computational efficiency, it assumes null grid current, thus sacrificing accuracy when the grid voltage approaches and exceeds the cathode voltage. In particular, we start by considering the general quadric surface equation:

$$k_{p2}V_{pk}^2 + k_{g2}V_{gk}^2 + k_2 i_p^2 + k_{pg}V_{gk}V_{pk} + k_{pk}V_{pk}i_p + \\ + k_{gk}V_{gk}i_p + k_p V_{pk} + k_g V_{gk} + k_1 i_p + k_0 = 0\,. \tag{8}$$

In order to exploit such an equation for modeling the plate current i_p, we set $k_1 = -1$ and $k_2 = k_{pk} = k_{gk} = 0$ for avoiding dependences on i_p cross-products or second-order powers, yielding

$$i_p = k_{p2}V_{pk}^2 + k_{g2}V_{gk}^2 + k_{pg}V_{gk}V_{pk} + k_p V_{pk} + k_g V_{gk} + k_0\,. \tag{9}$$

Considering the initial hypothesis on the grid current, after some algebra, we may write down the proposed quadric surface model as follows:

$$i_p = \begin{cases} i_{pk} & \frac{\mathrm{d}i_p(V_{pk})}{\mathrm{d}V_{pk}} \geq 0 \\ 0 & \frac{\mathrm{d}i_p(V_{pk})}{\mathrm{d}V_{pk}} < 0 \end{cases}\,, \tag{10}$$

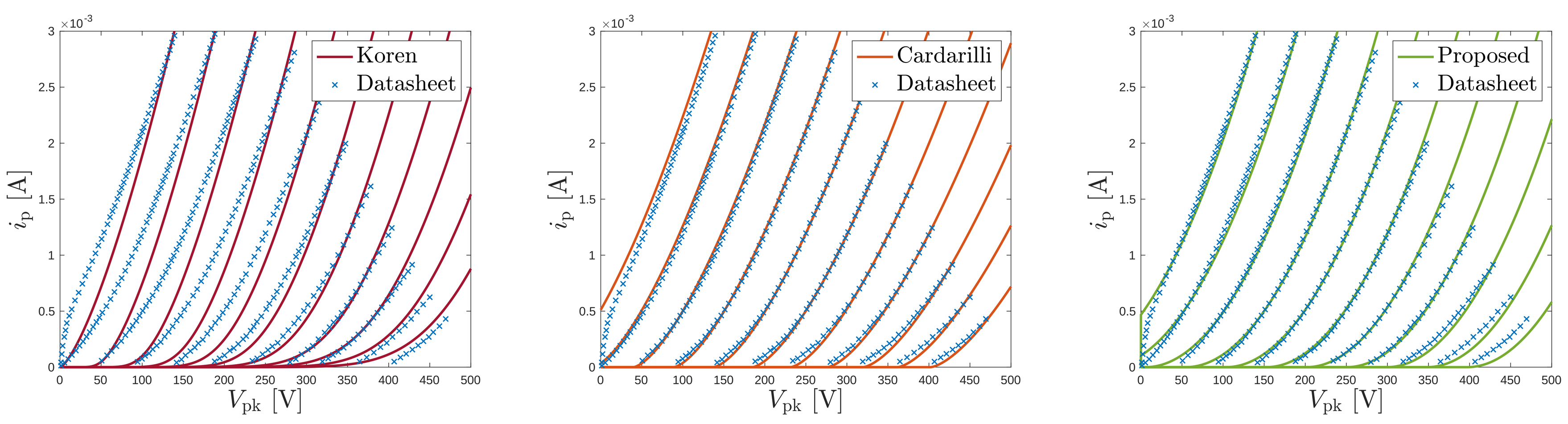
$$i_g = 0\,, \tag{11}$$

$$i_k = -i_p - i_g\,. \tag{12}$$

with

$$i_{pk} = k_{p2}V_{pk}^2 + \frac{k_{pg}^2}{4k_{p2}}V_{gk}^2 + k_{pg}V_{gk}V_{pk} + k_p V_{pk} + \frac{k_p k_{pg}}{2k_{p2}}V_{gk} + \frac{k_p^2}{4k_{p2}} = i_p(V_{pk})\,. \tag{13}$$

In any case, $V_{pk} \geq 0$ is also always assumed to hold true, as otherwise a region with negative static resistance would be introduced. In order to test the accuracy of the representation, we compare the $i_p - V_{pk}$ characteristics found on datasheets to that obtained by means of the proposed model. The proposed model maintain, in general, a comparable accuracy with respect to Cardarilli's.



3-port Wave Digital Implementation

The proposed model can be implemented following several different approaches. We implemented it as a 3-port element in the Wave Digital (WD) domain [4, 3, 5]. We name the ports between the p, g, and k nodes and node O as port 1, 2, and 3, respectively:

$$b_1 = \begin{cases} \frac{\sqrt{\Delta_1}}{\sqrt{2Z_1 k_{p2}\gamma}} - \frac{1}{4Z_1 k_{p2}\gamma^2} - \eta & \Delta_1 \geq 0 \\ a_1 & \Delta_1 < 0 \vee \frac{\mathrm{d}i_p(V_{pk})}{\mathrm{d}V_{pk}} < 0 \\ \frac{(Z_3 - Z_1)a_1 + 2Z_1 a_3}{Z_1 + Z_3} & V_{pk} < 0 \end{cases}\,, \tag{14}$$

$$b_2 = a_2\,, \tag{15}$$

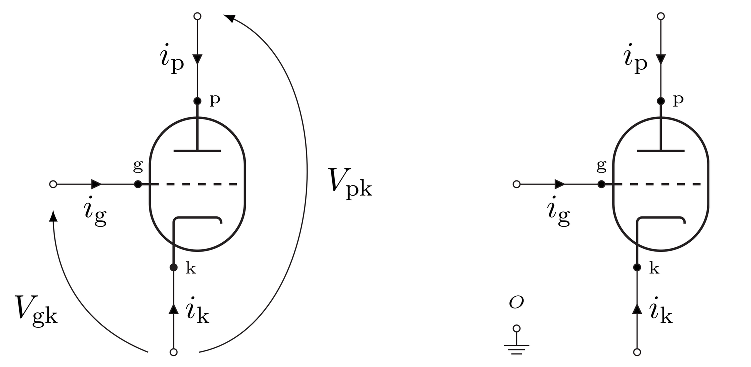
$$b_3 = a_3 + \frac{Z_3}{Z_1}(a_1 - b_1)\,, \tag{16}$$

where

$$\Delta_1 = \frac{1}{8Z_1 k_{p2}\gamma^2} + a_1 + \eta\,,\quad \gamma = 1 + \frac{Z_3}{Z_1}\left(1 + \frac{k_{pg}}{4k_{p2}}\right)\,,\quad \eta = \frac{\beta + \frac{\alpha}{2}}{k_{p2}\gamma}\,, \tag{17}$$

$$\alpha = k_p + k_{pg}\left(a_2 - a_3 - \frac{Z_3}{2Z_1}a_1\right)\,,\quad \beta = k_{p2}\left(\frac{1 - \frac{Z_3}{Z_1}}{2}a_1 - a_3\right)\,. \tag{18}$$

2 - Background on Triode Modeling



Koren Model

In [1], Koren presents a phenomenological model able to describe the behavior of triodes making use of a small amount of parameters. The model equations are designed such that the plate current $i_p > 0$ whenever the plate-to-cathode voltage $V_{pk} > 0$:

$$i_p = \frac{V_1{}^k}{k_{g1}}\left(1 + \mathrm{sgn}(V_1)\right)\,, \tag{1}$$

$$i_k = -i_p - i_g\,, \tag{2}$$

$$V_1 = \frac{V_{pk}}{k_k}\log\left(1 + \exp\left(k_k\left(\frac{1}{\mu_k} + \frac{V_{gk}}{\sqrt{k_{vb} + V_{pk}^2}}\right)\right)\right)\,. \tag{3}$$

Cardarilli Model

The triode model developed by Cardarilli *et al.* [2] relies on a unique approach that combines physical and interpolative techniques. This model replaces the constant parameters typically used in classical formulations for plate current i_p and grid current i_g with splines that depend on the grid-to-cathode voltage V_{gk}:

$$i_p = G\sqrt{\left(V_{gk} + \frac{V_{pk}}{\mu} + h\right)^3}\,, \tag{4}$$

$$i_g = \begin{cases} \frac{i_p}{1 + D\left(\frac{V_{pk}}{V_{gk} - V_{off}}\right)^K} & \text{if} \quad V_{gk} > V_{off} \\ 0 & \text{if} \quad V_{gk} \leq V_{off} \end{cases}\,, \tag{5}$$

$$i_k = -i_p - i_g\,, \tag{6}$$

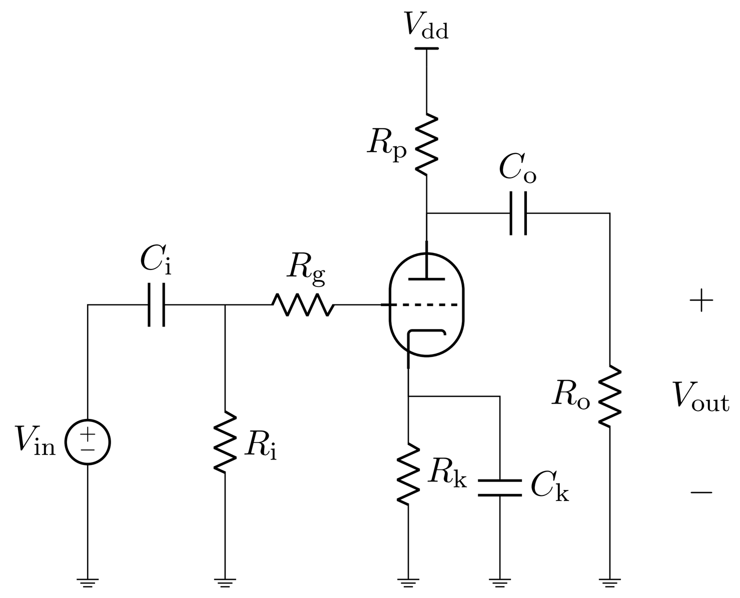
where

$$G = \sum_{i=0}^{3} G_i V_{gk}^i\,,\quad \mu = \sum_{i=0}^{3}\mu_i V_{gk}^i\,,\quad h = \sum_{i=0}^{3} h_i V_{gk}^i\,. \tag{7}$$

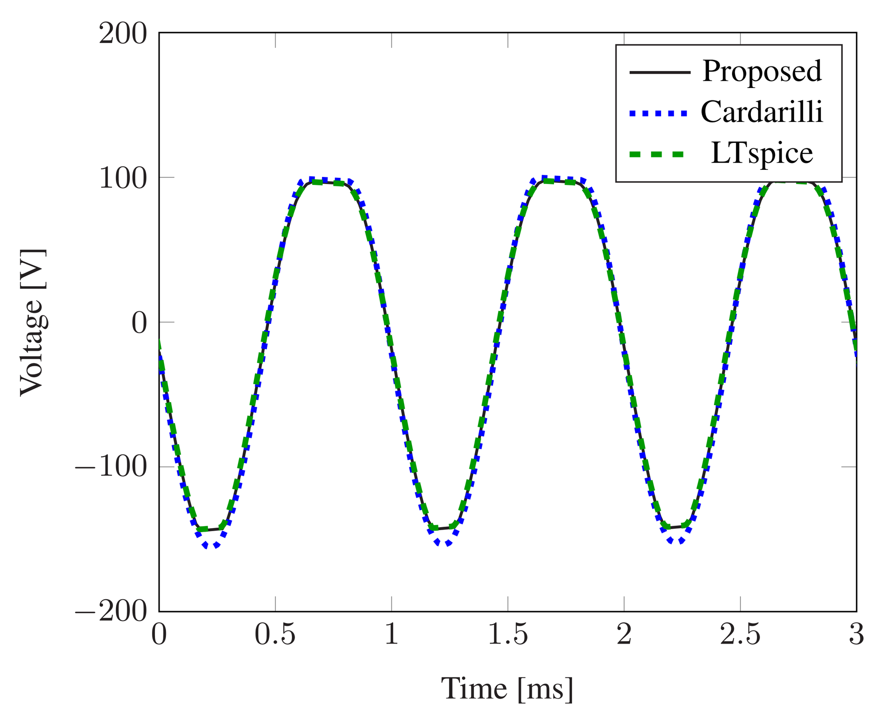
4 - Example of Application

Simulation of the Typical Gain Stage Found in Tube Amplifiers [3]

The following circuit is implemented with Wave Digital Filters. We realized the proposed model as a 3-port element.



We tested the proposed model by considering a sinusoidal input at 1 kHz with amplitude 2.5 V sampled at 44.1 kHz.



By considering as input a mono guitar audio signal of duration 16 s, we obtained a speedup of about $4.6\times$ over Cardarilli's approach.

References

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